An Introduction to Erlang B and Erlang C

If you make decisions about networks, PBXs, or call centres, you must understand these concepts

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"Hey, it's simple arithmetic! We get 3,200 calls a day. That's 400 calls an hour. Each call lasts three minutes, so each person can handle 20 calls an hour. So we'll need 20 incoming lines and 20 people to answer the phones."

Does that sound familiar?

Many telephone and call centre decisions are made using just that logic. It's simple, and it's clear — and it's absolutely wrong. The problem can be summed up in three words: *Calls Bunch Up*.

If you get 3,200 calls in an eight-hour day, and your calls follow a typical distribution pattern, 550 or 600 of them will arrive during the busiest hour of the day. And then they will bunch up during that hour, leading to times when the phones are going crazy and times when many lines are free.

Then you have to take into consideration the fact that no one can handle 20 three-minute calls every hour without burning out, and that there is usually post-call work to do in addition to talking to callers.

In short, the "simple arithmetic" approach will result in too few trunks, too few people — and too many very unhappy callers.

The alternative — the only approach that actually works — uses mathematical formulas that form the basis of the discipline called "traffic engineering." Some of them are easy for anyone to use; others require expert support, or at least specialized software. If you make configuration decisions about networks, PBXs, or call centres, you must understand the basic concepts involved, whether or not you plan to do the calculations yourself.

It Started in Denmark

A.K. Erlang, 1878-1929

All modern methods for optimizing networks have their roots in work done by Agner Krarup Erlang, a scientist who joined the Copenhagen Telephone Company in 1908. He set out to solve the key problem in telephone network design: how many trunks are needed to carry a given amount of calling?

Imagine a village in which every home has a telephone connected to a local switch. How many trunks should the phone company install between that switch and the one in the next village? Erlang saw that there was no absolutely correct answer. Rather, there is always a trade-off between service and cost. In the case of the village, there are two extreme options, neither of which is acceptable:

- Provide just one trunk, and let callers wait until it's available. The cost is low, but the service is unacceptably poor.
- Provide one trunk for every local phone line, so no call is ever blocked. The service is excellent, but the cost is much too high.

The problem was to convert that insight into hard numbers that would allow network planners and accountants to evaluate each possibility on the curves between low and high cost, poor and excellent service.

To do this, Erlang conducted the world's first detailed studies of telephone traffic, and then developed mathematical formulas to evaluate the trade-offs. His work was enormously influential worldwide. One Bell Labs researcher taught himself Danish just to be able to read Erlang's papers in their original form. In 1946, the International Consultative Committee on Telephones and Telegraphs (CCITT) honoured him by adopting the name "erlang" for the basic unit of telephone traffic.

Basic Concepts

Until recently, anyone using traffic engineering techniques had to either master the mathematics, or learn to use thick books of tables based on them. Personal computers changed all that, first by making traffic programs widely affordable, and now by making them available on the Internet. Several sources are described in the box on page 7.

But remember the phrase, "garbage in, garbage out." Before you jump to your browser, be sure you understand some key concepts.

Erlang: The basic unit of telecom traffic intensity. Strictly speaking, an erlang is what mathematicians call a "dimensionless unit," representing continuous use of one circuit. However, since a single circuit used continuously carries 60 minutes of calling in one hour, one erlang is usually defined as 60 minutes of traffic. If you receive 300 two-minute calls in an hour, then you received 600 minutes, or 10 erlangs, of traffic in that hour.

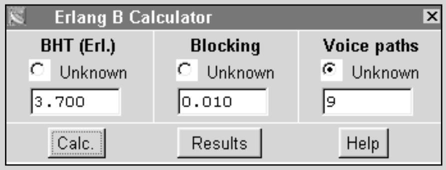
(For many years, AT&T and Bell Canada insisted on measuring traffic in CCS — 100 call seconds — instead of the more convenient but not-invented-here erlang. If you have data in CCS, divide the numbers by 36 to get erlangs.)

Busy Hour: Because "calls bunch up," all traffic planning has to focus on peak periods. It isn't acceptable to provide excellent service most of the time and terrible service just when customers want to make calls. Most commonly, we take the busiest hour of each day for five or 10 days during the busiest time of year, then calculate the average of those hours' traffic load. That "Average Busy Hour" figure is used to determine the maximum number of trunks or people needed.

Server: Something that handles calls. For example, in a call centre situation there are two kinds of servers: the trunks

Finding Erlang On-line

Software has replaced the thick books of tables that used to be basic equipment for anyone involved in optimizing telephone networks. The following are some useful resources. *Telemanagement* has not tested all of these products, and does not endorse any of them. All prices are in U.S. dollars.



■ **Erlang Calculator 1.0**: An Erlang B program for Palm computers. This simple freeware program displays three blanks: Fail Rate (Blockage), Traffic, and Lines. You fill in two and it calculates the third. Available from various sources, including *www.palmspot.com*, *www.visorvillage.com,* and *www.palmblvd.com*.

■ **Westbay Engineers**: A UK company that develops and sells traffic calculation software, ranging from *Erlang for Excel* ($80) to complex network and call centre modeling tools. Their website features the best on-line traffic calculators we've seen, including Erlang B, Erlang C, and call centre staffing. They also have an easy web address: *www.erlang.com.*

■ **HTL Telemanagement**: *Turbo Tables* ($595) was the first program to add traffic formulas to Excel, and remains one of the best. HTL focuses on call centre planning tools, using Hills A, a proprietary extension of Erlang C. *www.htlt.com*.

■ **Teleopti**: A Swedish firm that sells software for call centre management and optimization. Their website includes a rather complex Java-based *Call Center Wizard* for calculating trunk and staffing requirements. *www.teleopti.com*.

■ **Erlang-Software**: Another source for PC traffic software, this one in Australia. Their *Erlang-G* program is said to include six common traffic formulas. $40, with multiple-copy discounts. *members.iinet.net.au/~clark/index.html*.

■ **Certis Technologies**: Their *ErlangCalc 1.2* program is available in three versions: Standard (Erlang B only, $39); Pro (Erlang B, Extended Erlang B, Erlang C, Daily Traffic, $69); Deluxe (Erlang B, Extended Erlang B, Erlang C, Daily Traffic, Spreadsheet for Erlang B Batch-Processing, Graphical Presentation of Results, $89). Free trial versions are available. *www.certis.com/*.

■ **ClienTel**: Don't be dismayed when you find that the home page is in Dutch. Just click on *Online Call Center Assistant* for Erlang B and Erlang C calculators and a 24-hour call centre scheduling program. *www.clientel.nl/*.

■ **Maths.org**: Interested in the math behind Erlang's formulas? There's a tutorial, including some simple on-line calculators, at *plus.maths.org/issue2/dar/index.html.*

■ **Basic on-line calculators**. When you need quick answers, these non-commercial sites can be useful. Most use few graphics and download quickly, a major advantage.

www.dcss.mcmaster.ca/~qiao/publications/erlang/erlang.html
mmc.et.tudelft.nl/~frits/Erlang.htm
www.owenduffy.com.au/electronics/telecommunications.htm
www.cs.vu.nl/~koole/erlang.html
persoweb.francenet.fr/~hilleret/

that carry the calls, and the agents who answer them. With a voice mail or IVR system, the servers are ports.

Grade of Service: The probability that all servers will be busy when a call attempt is made. For example, on a trunk group: P.02 means that there is a 2% probability of getting a busy signal (being "blocked") when you have a given amount of traffic and a given number of trunks. In a call centre, the same number would mean that there is a 2% probability of having to wait to speak to a human.

Probability Formulas

There are many traffic formulas, appropriate to many different situations, but two of them, both developed by A.K. Erlang, cover the most common business telecom requirements.

Erlang B: This is the formula to use when a blocked call is really blocked — for example, when somebody calls your phone number and gets a busy signal or tries to access a tie trunk and finds it in use. It is built around three variables: Servers, Traffic, and Grade of Service. If you know any two of those, the formula will calculate the third one.

Erlang C: Use this formula when a blocked call is delayed — for example, when someone calls your call centre and must wait for an agent to take the call. It uses the same three variables, plus the average length of each call, to calculate the probability of being delayed and how long the delay is likely to be.

These formulas only work if you have a large number of independent sources of traffic. For example, 10 people making outgoing calls, with no incoming calls, will never need more than 10 trunks, no matter what the formula says! A mathematician will tell you that these formulas require "infinite sources," but in practice they work very well if there are at least 10 times as many possible sources (callers) as servers (trunks or agents).

Erlang B: The Easy One

The most common traffic engineering problem involves sizing a trunk group — how many trunks are needed to carry your toll-free calls, how many tie trunks between two offices, how many ports into your voice mail system, or some similar question. Erlang B handles that relatively easily, in four steps:

1. **Collect traffic data**. You need to know how much traffic will try to use the trunk group, each hour, for five or 10 business days. You may be able to use phone bills, call detail reports, carrier traffic studies, or even manual counts — or you may just have to make educated guesses. The objective is to produce an hour-by-hour spreadsheet, showing the number of minutes of traffic in each hour. Divide those numbers by 60 to get erlangs per hour.

Bear in mind that the traffic on your trunks may be greater than actual conversation time — you must allow for dialing time on outgoing calls, and for ringing time on incoming calls, for example.

2. **Determine the Average Busy Hour**. Select the busiest hour of each business day, total the traffic, then divide by the number of days.

3. **Choose a target Grade of Service.** In most cases, a target of P.05 is acceptable, P.10 is terrible, and P.001 is so good that most callers will never get a busy signal.

4. **Use Erlang B.** Calculate the number of trunks you need to carry that amount of traffic with your target Grade of Service.

At this point, you need to decide whether the answer is acceptable. Usually, that means deciding whether you can afford the number or trunks or ports required. If not, try reducing the number of trunks and see what the effect is on the Grade of Service. You may discover that the difference is negligible — or you may decide that you have to get a budget increase or find some way to reduce the amount of traffic. (Are those calls really necessary?)

The best thing about the various PC and Internet traffic calculators is that they make it very easy to do multiple calculations until you find a balance between cost and service you can live with.

Erlang C: More Difficult

Because Erlang B is so simple to use (insert two numbers, it calculates the third), many managers assume that Erlang C will be similarly easy. That's a mistake — even basic Erlang C calculations are difficult, and more complex ones can be daunting indeed.

Erlang C is most commonly used to calculate how long callers will have to wait before being connected to a human in a call centre or similar situation. This adds complexity in at least four areas.

Erlang B:

$$P = \frac{\frac{A^N}{N!}}{\sum_{x=0}^{N} \frac{A^x}{x!}}$$

Erlang C:

$$P(>0) = \frac{\frac{A^N e^{-A}}{N!} \frac{N}{N-A}}{1 - P + \frac{A^N e^{-A}}{N!} \frac{N}{N-A}}$$

1. **What's included in call times?** In a queuing system, traffic includes not just "conversation minutes," but also the time agents spend doing "post-call work" related to that conversation. Gathering accurate data on this can be much more difficult than just looking at a traffic study or a toll-free service bill. In theory, your ACD reports provide the information — but that only works if everyone has been pressing the right buttons at the right times.

2. **What's meant by delay?** The "Average Delay" might be the average of all calls, including all the calls that didn't wait at all, or it might be the average of calls that actually experienced a delay. The latter figure is usually more useful, but you must be very clear which one you are concerned with. Also, using averages can conceal situations in which most delayed calls wait only a few seconds, but some experience very long delays. That can be a serious customer service problem, even if the averages look good.

Most call centres summarize their delay objectives in a phrase like "answer 80% of calls within 20 seconds," but getting from the Erlang C formula to that result can be difficult.

3. **What's the hour-by-hour load?** With trunks, you don't have the option of adding or removing circuits every hour, so you must install and pay for the number needed under peak load conditions. With people, the peak period determines maximum staffing, but you also need to calculate staff requirements for other times, and plan staff scheduling accordingly. This usually means doing separate calculations for each half hour period in every week you operate.

And bear in mind that the Erlang C forecast only tells you how many people must be answering phones at any given time. That's quite different from the number you must schedule to work each day, since almost no one can be on the phones 60 minutes an hour.

4. **How does waiting time affect trunk load?** The time a caller spends on hold listening to music adds to your trunk traffic — you can save money by having fewer agents to answer calls, but that may require adding trunks and/or increasing your toll-free service bill. You may have to do several iterations to determine the optimal mix of trunks, people, and delay.

These issues mean that anyone doing anything more than the simplest delay calculations — above all anyone doing regular staff and configuration planning for a call centre — should consider buying specialized call centre planning software. All such programs — despite the fact that every one claims to be absolutely unique — are ultimately based on A.K. Erlang's formulas from 90 years ago.

If you are comfortable with the math and at creating spreadsheets, consider buying one of the commercial programs that adds traffic formulas to Excel, making it much easier to do multiple "what if" calculations.

Every telecom professional should have at least a general familiarity with Erlang B and Erlang C. However, whether you use them yourself or evaluate reports produced by others, always keep three things in mind.

- Using math is no substitute for using your head. If the data you plug in to the formulas isn't valid, the answers won't be any good either.
- All formulas make assumptions that simplify reality. Erlang B assumes that callers who receive busy signals won't immediately try again. Erlang C assumes that delayed callers will wait on hold indefinitely. For low blockage rates and short queues, those assumptions don't cause a problem — but when your service level is poor, they can give misleading results.
- All traffic formulas calculate probabilities, not absolutes. Erlang B and C predict what will happen, on average, over many hours with similar traffic. Your actual experience in any specific hour can be quite different.

Related Reading

- *Call Center Management on Fast Forward*, by Brad Cleveland and Julia Mayben, includes a useful plain-English discussion of using Erlang C for call centre planning. Order from *www.incoming.com*
- *Tables for Traffic Management and Design*, by Ted Frankel, seems to be the only book of traffic tables still in print. Lee Goeller's introduction and Frankel's first three chapters constitute an excellent brief course in traffic engineering concepts. Order from *www.abcteletraining.com.*
- Cisco's website has useful tutorials. Go to *www.cisco.com* and type "Traffic Engineering" into the search box.

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